

A fully symbolic procedure reduces a large class of elliptic integrals to symmetric standard integrals, which can be computed numerically by an efficient method based on the duplication theorem and a polynomial in elementary symmetric functions.

The reduction is performed in two stages. In the first stage a general integral is reduced to a set of basic integrals, and an integral table containing some results of the first stage is presented. In the second stage the basic integrals are expressed in terms of symmetric standard integrals.

The integrals to be reduced have the form

$$I(\mathbf{m}) = \int_y^x \prod_{i=1}^h (a_i + b_i t)^{-1/2} \prod_{j=1}^n (a_j + b_j t)^{m_j} dt = \int_y^x \frac{\rho(t)}{s(t)} dt, \quad x > y, \quad (1)$$

where

$$s(t) = \prod_{i=1}^h (a_i + b_i t)^{1/2}, \quad \rho(t) = \prod_{j=1}^n (a_j + b_j t)^{m_j}, \quad n \geq h, \quad (2)$$

and m_j is an integer. The a 's and b 's may be complex, but the integral is assumed to be well defined, possibly as a Cauchy principal value. The line segment with endpoints $a_i + b_i x$ and $a_i + b_i y$, $1 \leq i \leq h$ is assumed to lie in the complex plane cut along the negative real axis, and square roots have their principal value. The integral is elliptic if $h = 3$ (the cubic case) or $h = 4$ (the quartic case). The reduction may involve also algebraic functions of the form

$$A(\mathbf{m}) = \frac{\rho(x)}{s(x)} - \frac{\rho(y)}{s(y)}. \quad (3)$$

We write

$$\mathbf{m} = (m_1, \dots, m_n) = \sum_{i=1}^n m_i \mathbf{e}_i, \quad (4)$$

where \mathbf{e}_i is an n -tuple with 1 in the i th position and 0's elsewhere. We define also $\mathbf{0} = (0, \dots, 0)$. In the cubic case the basic integrals are

$$I(\mathbf{0}); I(-\mathbf{e}_j), \quad 1 \leq j \leq n. \quad (5)$$

In the quartic case they are

$$I(\mathbf{0}); I(-\mathbf{e}_j), \quad 1 \leq j \leq n; I(\mathbf{e}_i), \quad 1 \leq i \leq 4. \quad (6)$$

Basic integrals of type $I(-\mathbf{e}_i)$, $1 \leq i \leq h$, are not linearly independent, nor are those of type $I(\mathbf{e}_i)$, $1 \leq i \leq 4$.

The first stage of reduction and the resulting table of integrals are simplified by defining

$$\hat{I}(\mathbf{m}) = I(\mathbf{m})/B, \quad \hat{A}(\mathbf{m}) = A(\mathbf{m})/B, \quad B = \prod_{j=1}^n b_j^{m_j}, \quad r_{ij} = \frac{a_i}{b_i} - \frac{a_j}{b_j}, \quad (7)$$

where r_{ij} is assumed to be finite and nonzero so that different linear factors in the integrand of $I(\mathbf{m})$ are not proportional. $\hat{I}(\mathbf{m})$ is first reduced to integrals in which \mathbf{m} has at most one nonzero component, and the reduction is then completed by using two recurrence relations. See J. Symbolic Comp. 28(1999)739-753 for the corresponding relations in unsimplified notation and for the expression of basic integrals in terms of symmetric elliptic integrals.